Revisiting basic assumptions in mathematics teacher education

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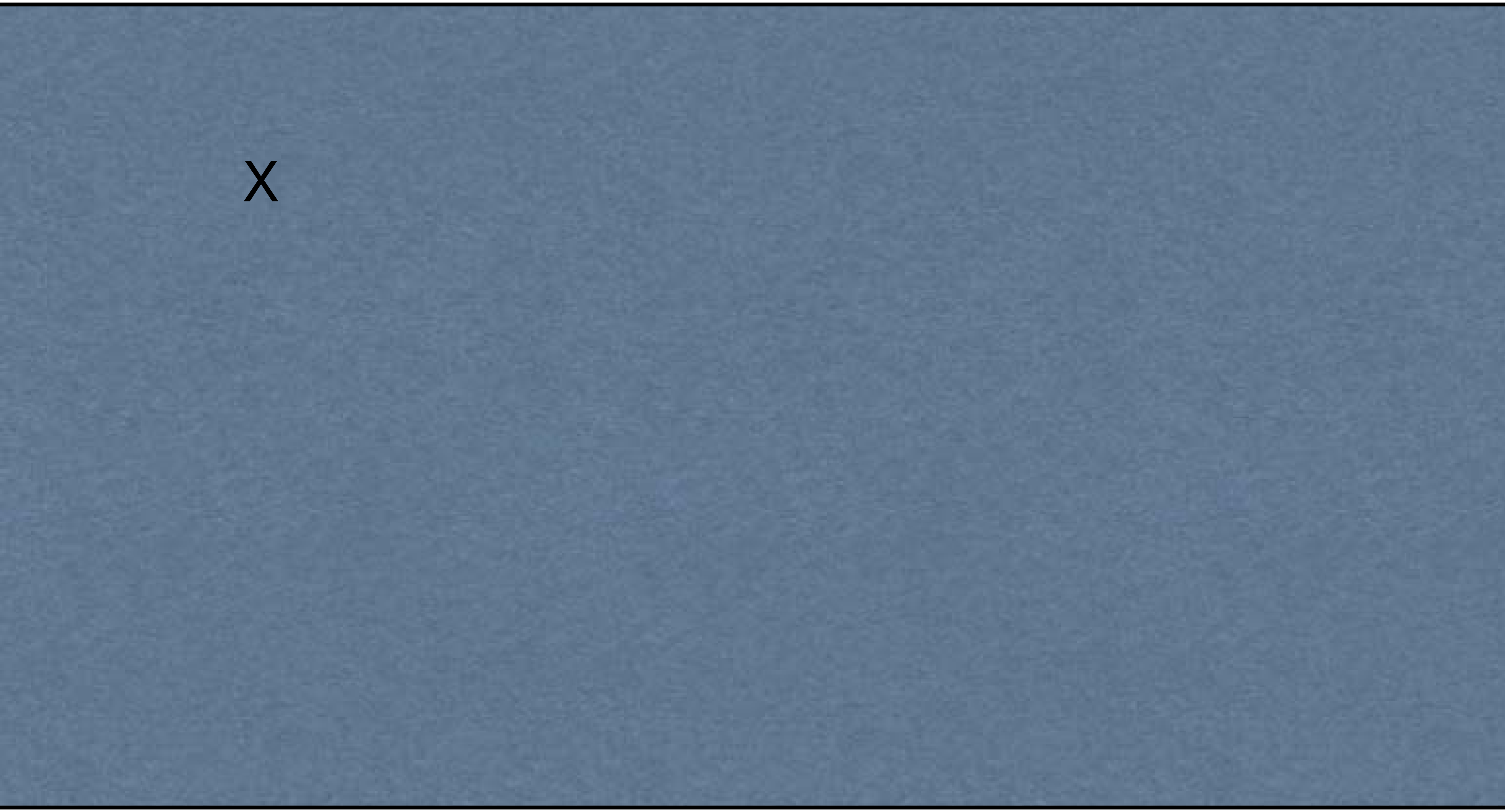
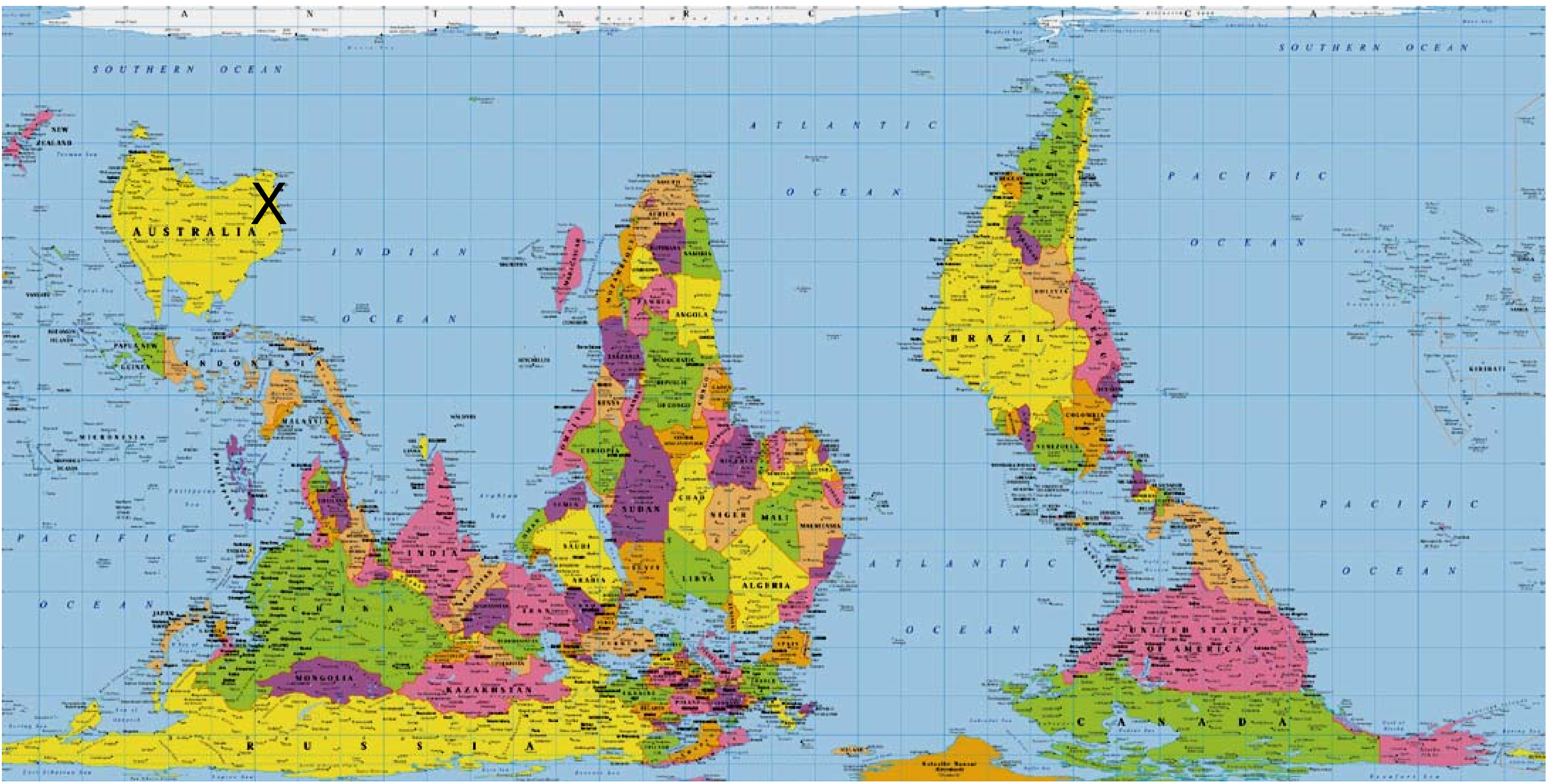
Goals in teacher education

- to improve/enhance teachers’ personal understanding of mathematics
- to examine/introduce the variety of students’ possible understandings or misunderstandings of mathematics



Imagine there is a world map in the above rectangle

Mark Vancouver, BC Canada on the map



A N T A R C T I C A
SOUTHERN OCEAN
NEW ZEALAND
AUSTRALIA
X
INDIAN OCEAN
ATLANTIC OCEAN
PACIFIC OCEAN
PACIFIC OCEAN
PACIFIC OCEAN
ATLANTIC OCEAN
BRAZIL
COLOMBIA
VENEZUELA
PAPUA NEW GUINEA
INDONESIA
MALAYSIA
ANGOLA
ETHIOPIA
SUDAN
CHAD
NIGER
MALI
LIBYA
ALGERIA
SAUDI ARABIA
INDIA
CHINA
JAPAN
MONGOLIA
KAZAKHSTAN
RUSSIA
UNITED STATES OF AMERICA
CANADA

Vancouver?

What is your reaction to the following statements?

- The sum of the interior angles in a triangle ABC in 280 degrees
- The graph of a function y=x is a parabola
- A number is divisible by 5 if and only if the sum of its digits is divisible by 5

What is your reaction to the following statements?

- The sum of the interior angles in a triangle ABC in 180 degrees
- The graph of a function $y=x^2$ is a parabola
- A number is divisible by 3 if and only if the sum of its digits is divisible by 3

And how about these?

- The sum of the interior angles in a triangle is always 180 degrees
- A graph of a function y=x is a straight line
- A number is divisible by 3 if and only if the sum of its digits is divisible by 3

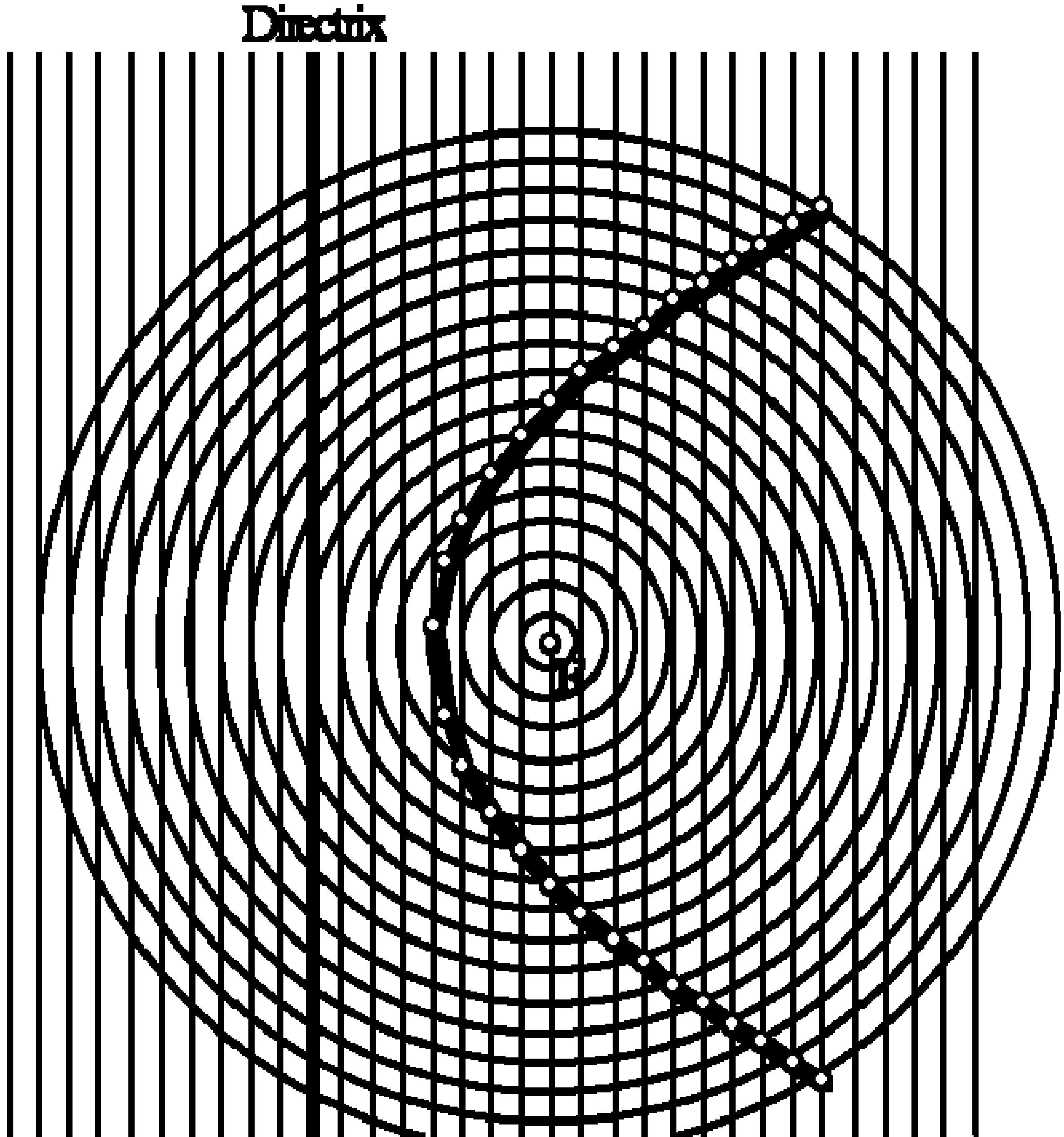
Look again

- The sum of the interior angles in a triangle ABC in 280 degrees
- The graph of a function y=x is a parabola
- A number is divisible by 5 if and only if the sum of its digits is divisible by 5

Look again

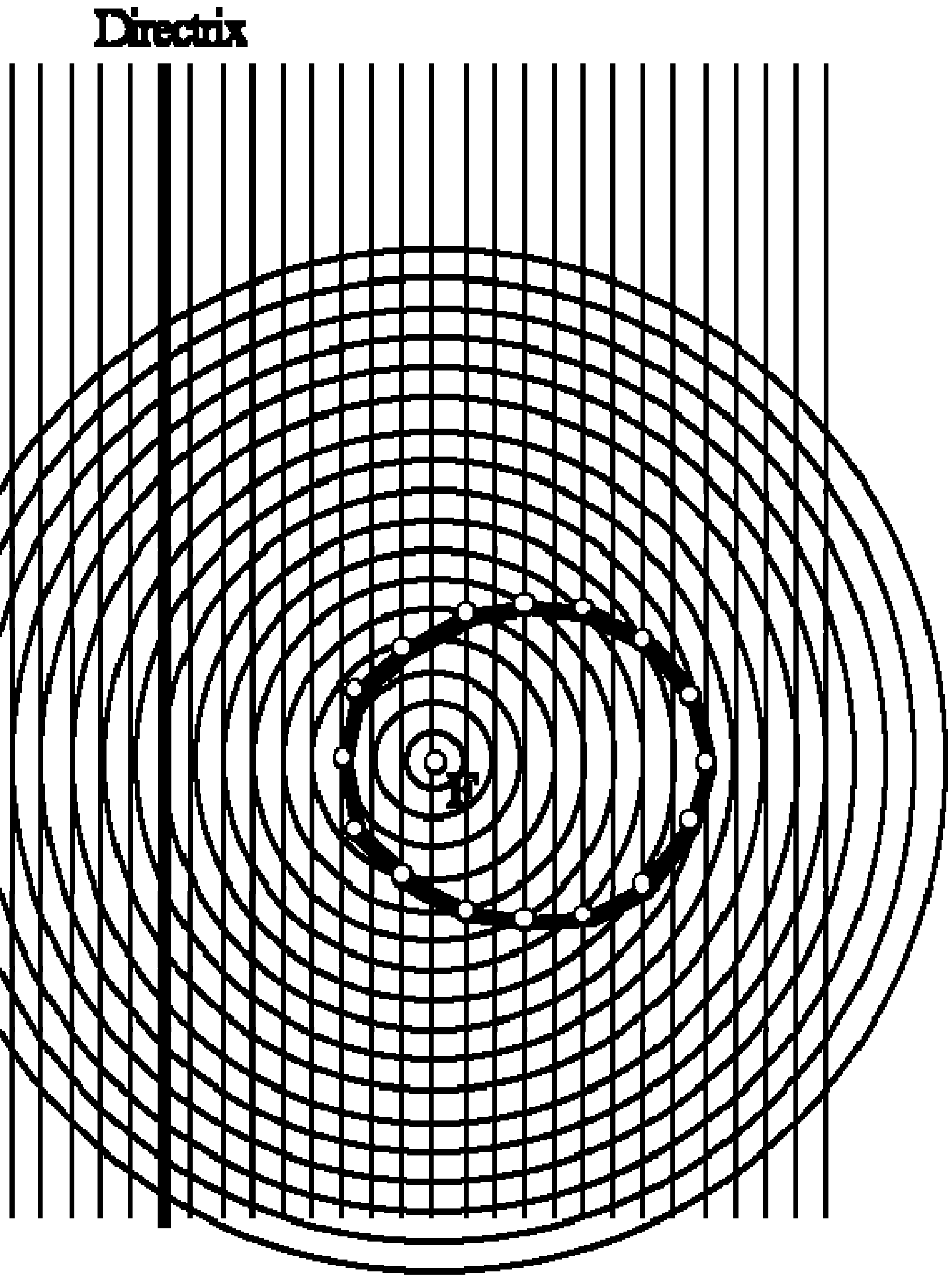
- The sum of the interior angles in a triangle ABC in 280 degrees (possible, on a sphere)
- The graph of a function y=x is a parabola (indeed, in focus-directrix coordinate system)
- A number is divisible by 5 if and only if the sum of its digits is divisible by 5 (indeed, in base 6)

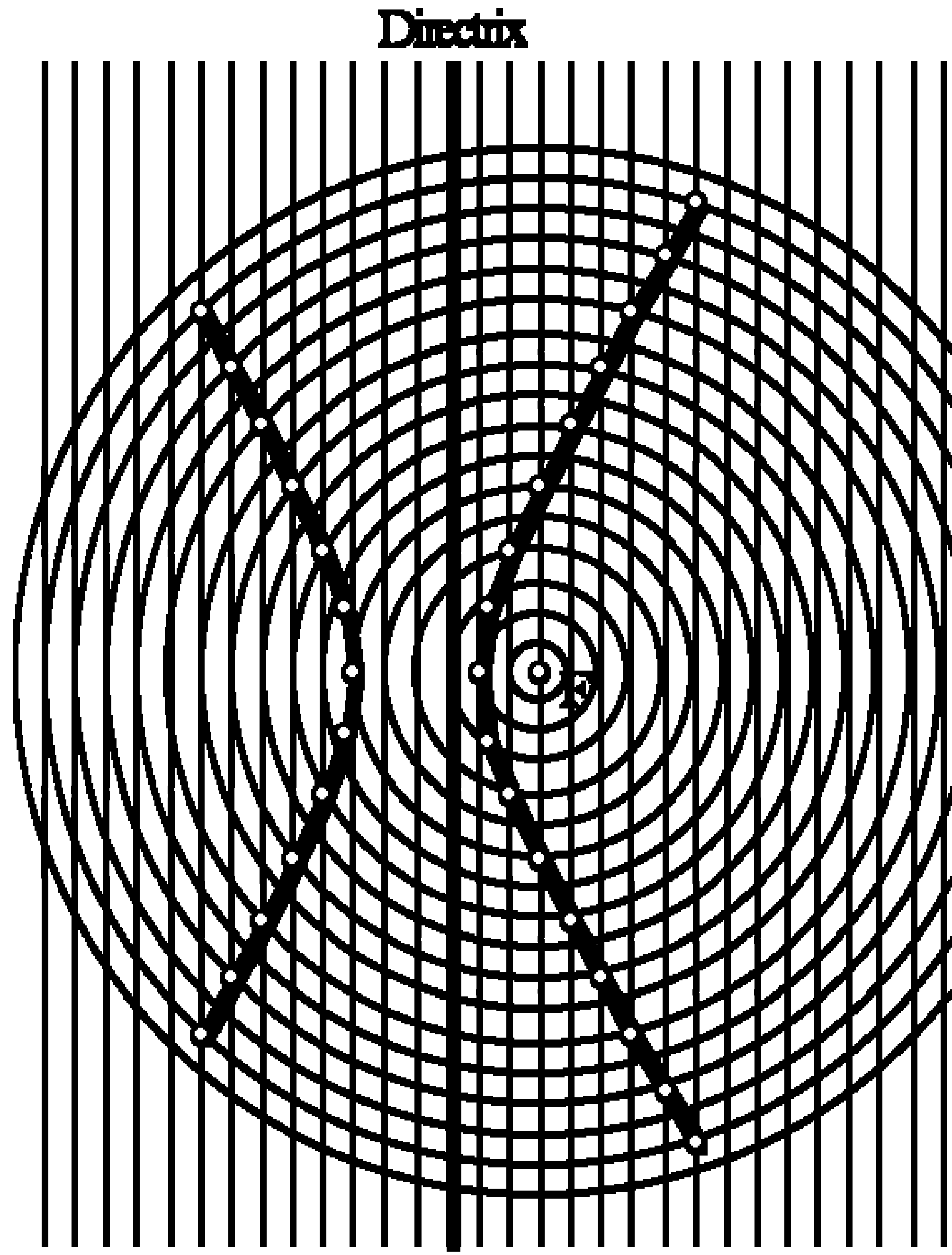
Directrix coordinates



y=x

Directrix coordinates





Examples that “shake” our assumptions

- conventions
- shared understandings
- unintended constraints

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Conventions

- Base ten representation
- Cartesian coordinates
- Euclidian geometry (on a plane)

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Multi-base arithmetic:

Dines | "New Math"

"if compared with mathematics resulting from pondering more profoundly the subject matter and its relations to reality, unorthodox positional systems are a mere joke"

Freudenthal (1983, p. 132)

"it is a good didactics to motivate pupils by jokes, and an unorthodox positional system may even be a good joke".

Working in bases other than 10

- Counting
- Conversion
- Operations

count in
base 4

1 , 2 , 3 10, 11, 12, 13, ,
20, 21, 22, 23, 30,
31, 32, 33,
, , 100, 101,
102, ...

Add in
base 5

$$\begin{array}{r} 33 \\ +14 \\ \hline 102 \end{array}$$

Multiply in
base 5

$$\begin{array}{r} 33 \\ \times\ 14 \\ \hline ? \end{array}$$

convert 12.34_{five}

$1 \times 5 + 2 \times 1 + 3 \times (1/5) + 4 \times (1/25) = 7 + 19/25$

$= 7.76$

convert 12.34_{five}

1x5 + 2x1 + 3x(1/5) + 4x(1/25) = 7+19/25
= 7.76

1x5 + 2x1 + 3x(1/5) +
4x(1/50)

$12_{five} = 7_{ten}$; $34_{five} = 19_{ten}$

$12.34_{five} = 7.19_{ten}$

7+

34/25

play New Math

FOR TEACHERS…

From challenging extensions or extra-curricular activities
to

extending the boundaries of teachers' example spaces

gaining renewed understanding of (automated) processes
– such as multi-digit addition or assigning place values –

embedding the conventional in an extended schema

gaining insight on student's difficulties

Examples that "shake" our assumptions

- conventions
- shared understandings
- unintended constraints

Shared common understandings

Grandma baked 12 cookies for 3 of her grandchildren.

How many cookies will each child get?

280 students of ABC elementary school will go in a field trip by buses.

There are 40 seats on a bus.

How many buses are needed?

Mary had 4 blouses, 3 skirts and 2 jackets.

How many outfits can she make?

Shared common understandings

I ate healthy foods for 2 weeks and lost 7 pounds. How many pounds will I lose if I eat healthy foods for 20 weeks?

Jake bought a twelve-pack of beer and paid $10.44. He then decided he needed two more cans of beer. How much will it cost him?

FOR TEACHERS...

From simple exercises, drill and practice

to

raising awareness of what is implicitly taken for granted and can be an obstacle for a learner

Examples that "shake" our assumptions

- conventions
- shared understandings
- unintended constraints

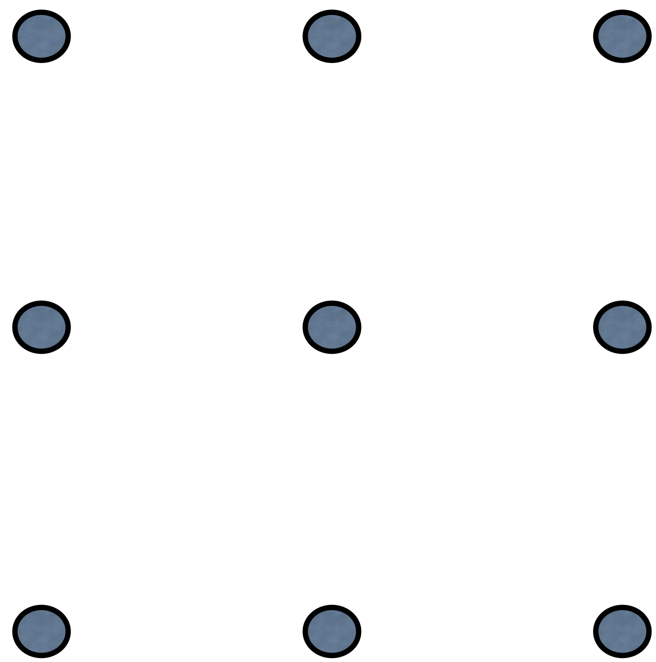
Can you cut a cardboard square
into 10 squares,
using all the material?

Can you plant 4 trees such that
there is the same distance
between any 2 of them

play
movie

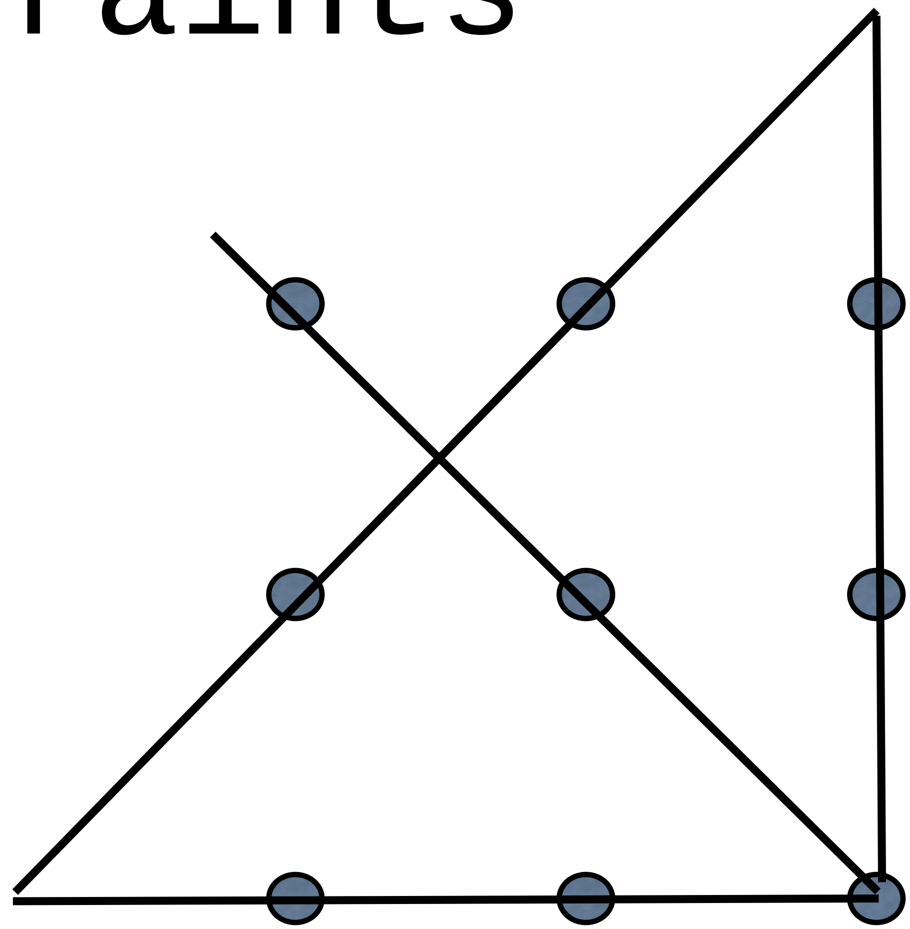
Assumptions as constraints

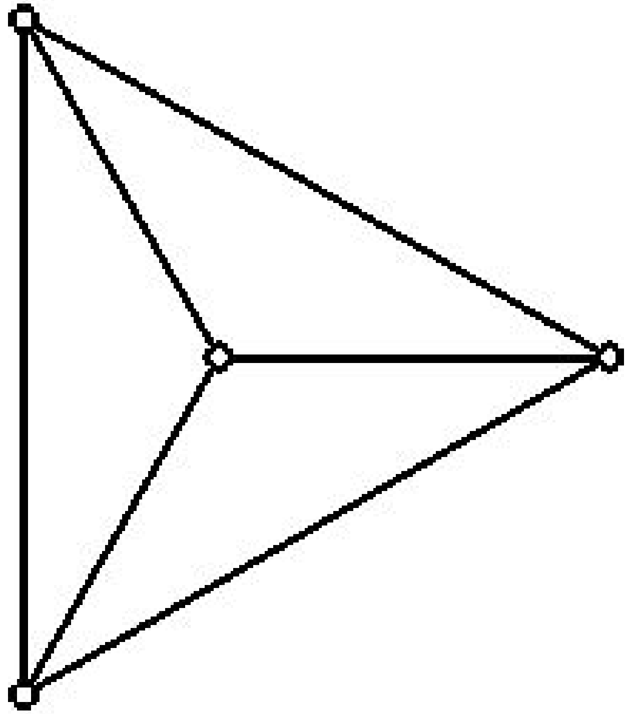
famous 9 dots



Assumptions as constraints

famous 9 dots





Can you plant 4 trees such that there is the same distance between each 2 of them

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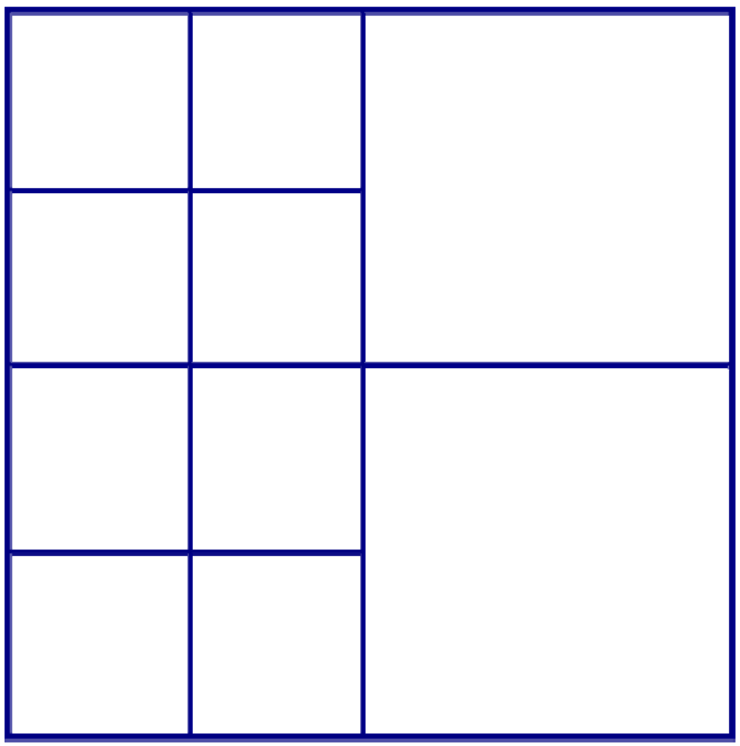

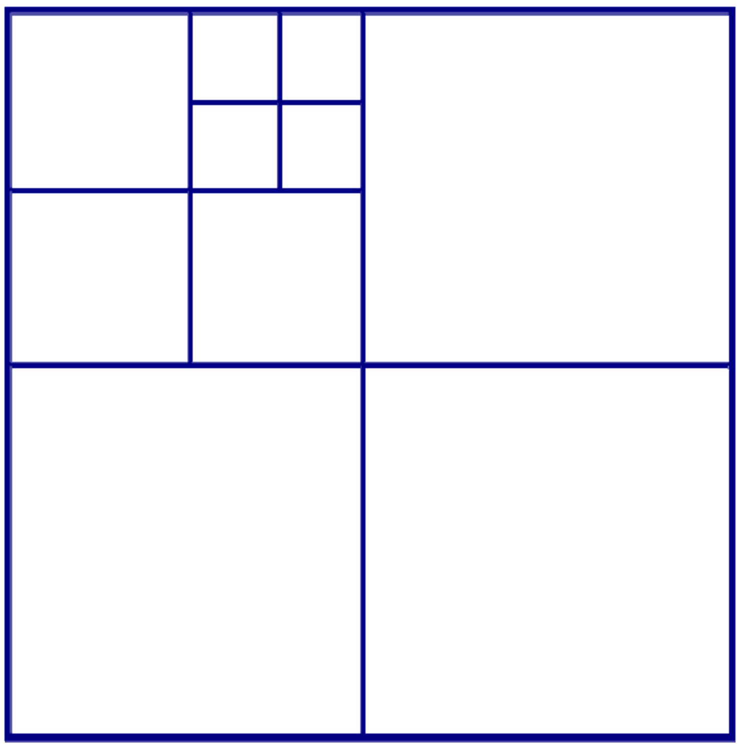

FOR TEACHERS...

From riddles and brainteasers

to

identifying constraints in human thinking
in support of problem solving

Schema

“to understand something means to assimilate it into an appropriate schema” (Skemp, 1973)

How can one understand better what has been already understood, that is, assimilated?

“to understand something better means to assimilate it in a richer or more abstract schema” (Zazkis, 20XX)

How can this happen?

Black swans


THANKS