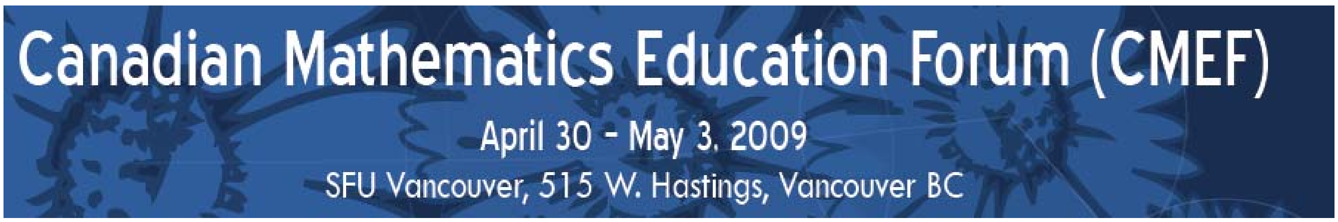
ONLINE LEARNING: COLLABORATIVE MATHEMATICAL EXPLORATION



Report Based on the Canadian Mathematics Education Forum (CMEF) Working Group Meeting and Online Group Discussions

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Background Information

The Online Working Group met in Vancouver in 2009, during the fourth meeting of the Canadian Mathematics Education Forum (CMEF). This series of meetings was first started in Quebec City in 1995, and then was followed by the meetings in Montreal in 2003, and in Toronto in 2005. The main organizers for the 2009 Canadian MEF have issued a call for proposals in 2007. Among those selected were four proposals that were linked to online learning or use of the Web. These were submitted by Geoff Roulet, Viktor Freiman, Jocelyn Dagenais, and Dragana Martinovic. There were others involved in these submissions—George Gadanidis was a partner in the Roulet submission, and Zekeriya Karadag and Adina Bardas were involved in Martinovic's proposal. The CMEF 2009 organizers suggested putting these four proposals together, which marked the initiation of the present Online Working Group.

During the preliminary online discussions, the group members have opted for a somewhat narrower focus than the title "Online Learning" hinted. Since the group wished to concentrate on using the Web/Internet to support collaborative mathematical exploration, the title *Online Learning: Collaborative Mathematical Exploration* was selected as more appropriate.

This report provides information about the work done by the *Online Learning: Collaborative Mathematical Exploration* working group during these two years of preparation for the CMEF09, as well as for the duration of culminating activities at the Forum.

Online learning, then and now

During the previous CMEF meeting in Toronto, the Online Working Group developed a White Paper with the intent to "produce a statement concerning the teaching and learning of mathematics via the Web" (Gadanidis, Graham, McDougall, & Roulet, 2002, p. 3). In the context of changes in mathematics education, the authors presented a workshop with the following vision statement:

> We want students to be immersed in a mathematics culture that gives them an opportunity to: a) learn, use, and refine inquiry, investigation, experimentation and problem solving processes, and b) develop the tools/skills/habits of a life-long

> learner, learn significant concepts and procedures (with understanding) that they can then use in an integrated, authentic fashion to conduct inquiries, experiments, investigations and problems. (p. 4)

The participants were asked to discuss characteristics of on-line environments which would support such mathematics learning. Apparently, the school boards were slow to incorporate online courses, and it was mainly upon individual, 'innovative' mathematics teachers to incorporate online activities and develop 'pedagogically sound' online resources. It was maintained that mathematics educators need to think about, discuss, and clearly describe the types of Web-based mathematics learning environments required. The White Paper concluded with a plethora of recommendations for the professional development of teachers, including pre-service programs, development of online resources, training, etc. (see Figure 1).

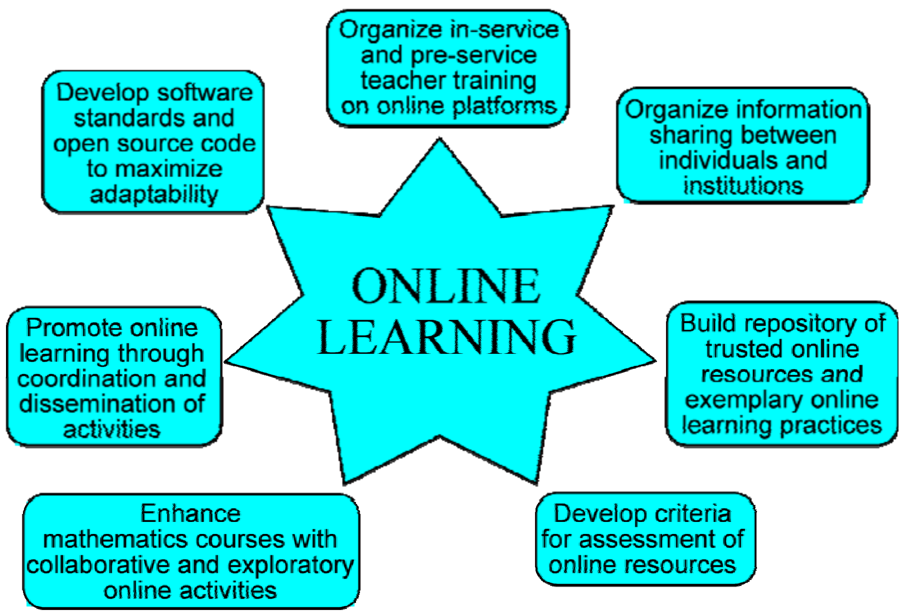

Figure 1: Summary of recommendations given at the 2001 Online Learning meeting

How did we form our group?

Building on the CMEF 2001 meeting on Online Learning in which three of our present working group members participated, namely Gadanidis, Roulet and Martinovic, the 2009 Working Group on Online Learning defined its new objective as:

> Investigate online opportunities to learn mathematics, especially those that are exploratory and interactive in nature. Determine ways in which the presentation

and communication capacities of the participatory Web 2.0 support the collective construction of mathematical knowledge, by focusing on the three key terms: *online*, *collaboration*, and *exploration.*

The online environments encompassed in this objective are the environments that allow users to share their work and ideas as well as to annotate and to comment on the others' work and ideas. The theoretical framework for students to learn mathematics through sharing their work and ideas is based on the idea of *distributed cognition* conceptualized by Pea (1993) and Salomon (1993). Some examples of these environments are:

- *online white boards* (i.e., www.scriblink.com) for small groups to communicate mathematically and collaborate synchronously;
- *application sharing software* (i.e., Elluminate Live!, Adobe Connect) in conjunction with mathematics software on participants' computers to support synchronous work;
- *communication software* (i.e., e-mail, discussion boards, blogs, wikis) to allow students to asynchronously discuss mathematics and share products by attaching files from mathematics software; and
- *wikis/blogs* to allow users to discuss the use of mathematics learning objects and dynamic worksheets; and elaborate on the ideas drawn from the objects.

Besides collective construction of mathematical knowledge in online environments which is conceptualized as distributed cognition, some researchers and mathematics educators turn their attention to the new quality achieved through use of technology as an "extension of self" (Galbraith, 2006), humans-with-media (Borba & Villarreal, 2006), and instrumentalization (Artigue, 2002). These scholars suggest that students could use technology as a cognitive tool, which allows them to extend their cognitive abilities and to (re)organize their thoughts and knowledge. According to these authors, the learning outcome of this new learning is a product created beyond the limits of human abilities as well as machine abilities; rather, it extends both. In addition, "The dynamic media co-acts with the user, so the user can guide or be guided by the software environment" (Moreno-

Armella, Hegedus, & Kaput, 2008, p. 109). As this co-action becomes more dynamic, the mathematics becomes more personalized and hence owned. In the complex networked environment (e.g., in online application sharing software) “agency and cognition are more distributed and the nature of the mathematical activities become more participatory” (p.109). Thus, this purposeful co-habitation of humans and technology supports collective, yet personalized co-construction of mathematical knowledge though collaboration and exploration.

Furthermore, as suggested by Borba and Villarreal (2006) and Moreno-Armella, Hegedus, and Kaput (2008), visualization is at the heart of this new type of learning. Moreno-Armella, Hegedus, and Kaput (2008) argue that representational systems used in mathematics and mathematics education have been evolved from a static-inert stage to a dynamic continuous stage and that visual representation should be considered as a more effective representation than the symbolic one. Therefore, in addition to the collaborative and explorative characteristics of online environments, we included visual learning, visualization, and visual representation into our discussion.

Our group’s wiki space

In order to overcome the space- and time-related barriers induced by the working group members being dispersed across the several Canadian provinces and in view of what our mission is about, we agreed that we would set up a wiki to support our interaction prior to the CMEF. We also agreed that the conversation would be bilingual with everyone posting in either French or English depending upon their comfort with either language. With this in mind, we started our online discussions prior to the Canadian Mathematics Education Study Group (CMESG) meeting in 2008, briefly met during the meeting and continued working online.

By communicating mostly online, we used the wiki space to develop our themes, to display online resources, and to plan our culminating working group meeting in Vancouver. By doing so, we operated in the environment that we were to investigate for use in mathematics and that gave a very interesting, meta-level to our collaborative thinking. After we have met in Vancouver face–to-face and got to know each other better, we experienced the benefits of blended online and face-to-face collaboration and could speak of its advantages. This working group consisted of graduate students, secondary teachers and university professors. It was striking that all in participation during the CMEF 2009, had connection to the secondary level of schooling. However, our virtual participants were also elementary teachers.

In addition to the ideas emerging through our online and face-to-face discussions at the CMEF 2009 meeting, we issued a call to teachers who could not attend the CMEF because of prohibitive cost for their boards or time-related conflicts, to send their reflections about online learning in their schools. These reflections are compiled into themes and presented in the special section of this document.

Meetings during the CMEF2009

In the CMEF2009 meeting, the Online Learning working group met four times in three days. All meetings were conducted in a computer lab with PCs and the Internet access—the environment highly conducive for exploration and testing of online resources we discussed. The lab was equipped with a data projector and a white screen where the presenters showed their slides and the online examples that were then tested on individual computers and discussed by the working group. The text that follows describes each of the meetings and provides excerpts from the discussion.

First meeting, Friday, 10:30am - 12:30pm (Zekeriya Karadag)

In the first meeting, we started by comparing definitions of visualization cited by Borba & Villarreal (2006) and provided by the presenter:

<u>Definition 1.</u> Visualization in mathematics is a process of forming images (mentally, or with paper and pencil, or with the aid of technology) and using them with the aim of obtaining a better mathematical understanding and stimulating the mathematical discovery process. (Zimmermann & Cunningham, 1991, as given in Borba & Villarreal, 2006, p. 80)

<u>Definition 2.</u> Visual image is a mental scheme representing visual or spatial information such as concrete pictorial imagery, pattern imagery, memory images of formulae,

kinesthetic imagery, and dynamic imagery. (Presmeg, 1986a; 1986b, as given in Borba & Villarreal, 2006, p. 80)

The audience opted for the first definition as better aligned with the purpose of using visualization in mathematics education. This purpose was also associated with the goals of online learning for mathematics, i.e., to improve mathematical understanding and stimulate discovery in mathematics.

We discussed the evolution of mathematics representation as suggested by Moreno-Armella, Hegedus, and Kaput (2008). They introduce a symbol as "something that takes the place of another thing" (p. 100), which is reference field of the symbol. For Moreno-Armella, Hegedus, and Kaput, the symbolic structures allow humans to "think deeper" and to share their thoughts. Consequently, the symbolic representation unifies the dynamic world of its reference field. A choice of symbolization affects what humans can do with it and allows them to learn more about their thinking process. The authors argue that symbolic representation of mathematics was first started by writing on bones and stones. After millennia of evolving through stages—from *static inert*, *static kinesthetic/aesthetic*, *static computational*, and *discrete dynamic*, we are now in the *continuous dynamic* stage. This stage is characterized by possibilities for dynamic exploration, visual exploration, and increased conceptual understanding of mathematics phenomena as a result of these types of exploration supported by technology. Then, we moved on the examples collected from various resources.

Example 1. The first example was an animation downloaded from Dr. Mourat Tchoshanov's web site (http://dmc.utep.edu/mouratt/, see Figure 2). This web site contains a number of animations visualizing mathematical identities such as difference of two squares and perfect squares.

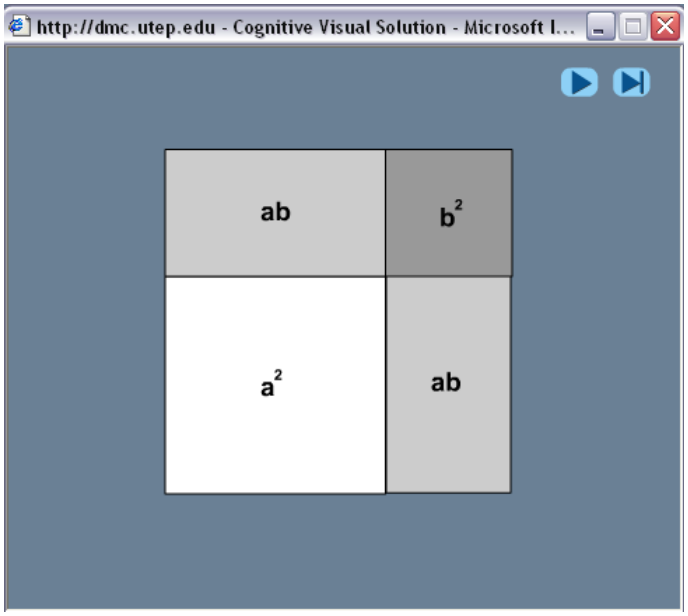

Figure 2: Prove that $(x+y)^2=x^2+2xy+y^2$

Some reflections on this example were as follows:

- Doing it by hand might be more helpful to students.
- My understanding is constrained by animation.
- What do you do with it is important.
- A child can get scared by it.
- This object visualizes the result of learning, not the process of learning.
- Disadvantage is that there is still an old algebraic relationship present. There should be more than one way to use the applet. Then students could share the interpretations.
- It is difficult to assess if it is useful. We are constrained by our thinking. It would be better to have a virtual reality glove and manipulate the object physically.
- This applet does not help with the basic misconception in algebra—students do not understand the concept of variable. Modification-rescaling-resizing the square could help.
- Advantage is that by restricting what can be done, the students are forced to look into relationships (have a narrow, pointed view). Sameness sometimes can help.

Example 2. The next two examples were virtual manipulatives from the National Library of Virtual Manipulatives web site, hosted by Utah State University (http://nlvm.usu.edu/en/nav/vLibrary.html, see Figure 3). We first examined an example related to the distributive laws in algebra and then the example related to patterning, known as Towers of Hanoi.

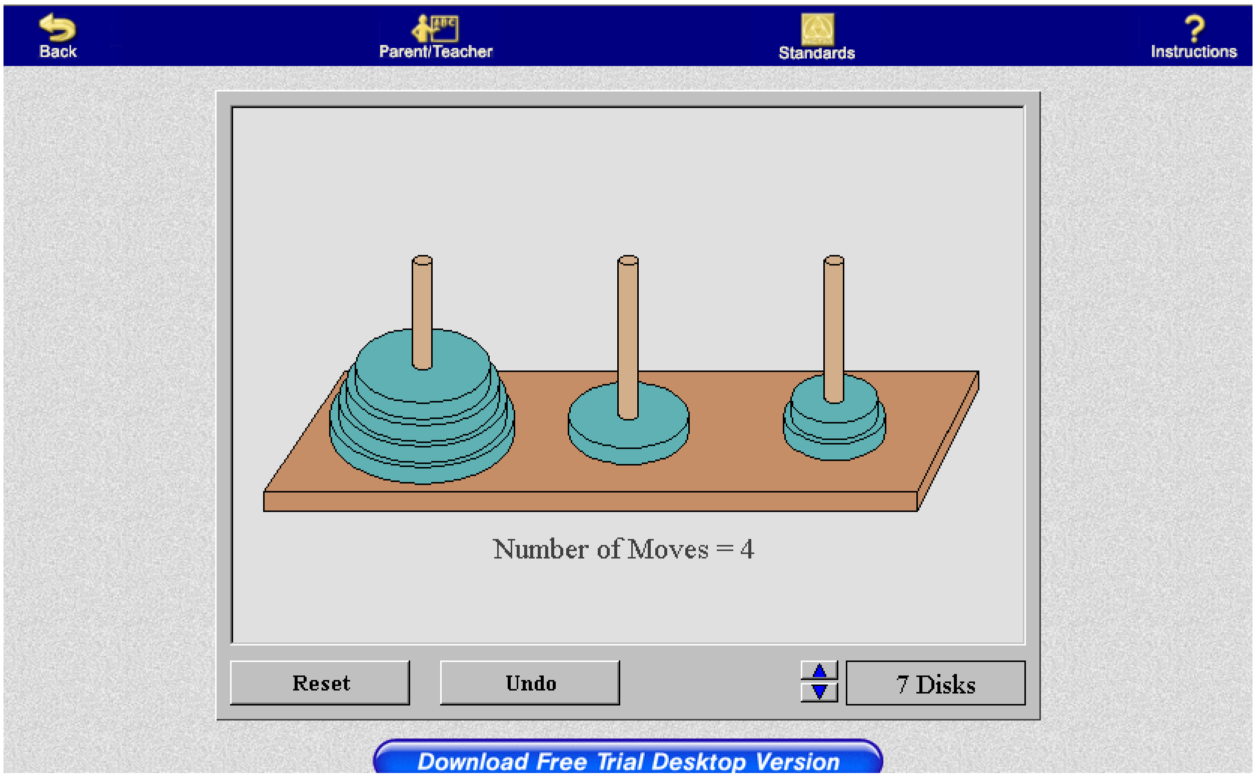

Figure 3: Virtual manipulative: Towers of Hanoi.

Some of the comments on the Towers of Hanoi example provided by the participants were as follows:

- It is not in curriculum, it is not in the textbook. If it is in the textbook, it is in a small print.
- Standardization is against technology. Technology brings about diversity and going into different directions.
- Each teacher can bring about different kind of knowledge for the students.
- Teachers do not have time to use extracurricular examples.

- Technology helps to bring about math teaching that is not based on formulae, but is based on logic and intellectual challenge. However, it seems that the system is not ready for implementing learning that is beyond laptops.
- Benefits are not in technology, but in teaching style. The culture of classroom is conducive to some technologies used earlier.
- Is technology enabler of change? Teachers need to find way to use technology in everyday practice. Some teachers made a switch.
- The change is also student-driven. Sometimes students want technology because there is less for them to do.
- Parents want the online presentation of mathematics problems to have a sense of accountability-see what their children need to do in order to succeed.
- We, as professors, do not know everything. We need to accept a position of teacher as a learner.
- Collect these stories. A lot of electronic resources are just one-minute events, but that is OK. These minute ideas are already meaningful.

Participants reflected on people who may not know formal mathematics but they can still produce beautiful, mathematically rich, patterns on carpets.

> Technology helps to bring about math teaching that is not based on formulae, but is based on logic and intellectual challenge.

Second meeting, Friday, 3:30pm - 5:00pm (Zekeriya Karadag)

In the second meeting, we explored GeoGebra, which is a dynamic, online, free, and open source learning environment for mathematics. The presenter illustrated how students can use GeoGebra to explore the relationships between concepts dynamically. For example, the group explored the relationship between the behaviour of a function and the slope of its tangent at a point on its graph. We observed the changes in the slope of the function at different segments on x-axis, such as increasing, decreasing, and at extremum points. Moreover, we examined the relationship between a trigonometric circle and the trigonometric functions defined by moving a point along the circle (see Figure 4).

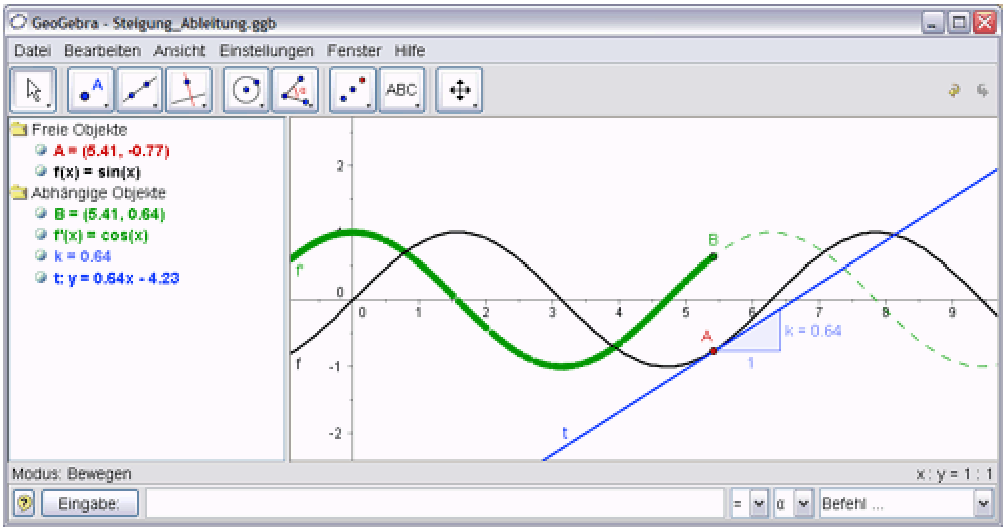

Figure 4: GeoGebra example. Sine and Cosine functions and their derivatives

The participants shared their experiences with similar tools. For example, Viktor Freiman said that he taught Grade 5 students by using Cabri and made powerful explorations. With technology, a lot of explorations can be done much earlier in schooling.

- With technology, the concept of infinity can be introduced easily and become intuitive.
- Going from triangle to quadrilateral to pentagon, …, with technology—these leaps could be made without the restraints of the curriculum.
- Students are asked if the sum of internal angles in triangle is always 180^{o}. They are asked to explore and think critically, to ask open ended questions. At that point kids think that all graphs are a straight line. Technology helps bring about more sophisticated mathematics.

The group had a discussion on how can students collaborate using GeoGebra? Students can be sent home to play with it. Ask them to do *PIE*: Predict➔Investigate➔Explain. After that they know what they are looking for. For many of these learning objects, it is obvious that they are mathematical. Saying “go and play with it and learn math,” does not motivate students. Their perspective of what it means to "learn math" may be quite different. Just say "go and play with it."

- How is "play" different from "explore"? Basic problem is that we do not talk about pedagogy. We need a separation between play and school, but let people do things naturally. Within a system, we limit the use of technology. With technology they may not know math better, but they will learn something.
- How do we assess thinking? Record the work in video format, add notes. One example is with the lesson provided as video. Definition of locus is introduced, students watch the intro, after that they discuss the shared experience.
- Do we need completely different assessment? "Describe what you see." How many times you have heard the teacher asking this question.
- Actually, there are examples in the curriculum that start with "describe," but teachers skip the text and move straight to the example. The usual path is to show, ask students to reproduce and do the test. Curriculum prescribes a content-oriented teaching. It is not allowing for alternative assessment.
- We are centred on a result not on a process.

Curriculum prescribes a content-oriented teaching. It is not allowing for alternative assessment.

Third meeting. Saturday 8:30am - 10:30am (Viktor Freiman)

We started the session by exploring the question: "What is online learning?" Some features of online learning that emerged through discussion are as follows:

- It is not face-to-face, but then, during videoconferencing people see each other.
- Students use computers as a communication and as a cognitive tool.
- It is distance learning in terms of online course, independent of space and time constraints.
- It is done individually and in community, with or without a teacher.

- Classroom is expanded, computers or any other device could be used, and computer network is an interface.
- Online learning is limited by availability of technology, access and knowledge.

Example 1. The presenter demonstrated an example from YouTube: *Smart schooling in Korea*. We observed that it is difficult to distinguish play from learning; Internet and video games are part of learning. It is instant society. Cyber cafés are open all night. Using Internet is addictive. But instead of restricting its use, we need to guide children to use it wisely. Technology is more than a tool; it is an actor in a society.

- Moreover, we focused on the use of video in online learning. It is emphasized that online learning impacts the public education system.
- Does it affect and change the ways we learn? It was noticed that all the positive attributes of online learning are related to communication, but in fact students are taken from face-to-face environment where they would most naturally communicate. It is not fair to compare online learning to classroom learning, nor is it fair to compare this kind of informal learning to the teacher-directed and formal learning. Online learners form a community, but by becoming part of such a community, do we lose the opportunity for debate and exposure to not "like-minded" people?
- Major characteristics of online learning are the enthusiasm and freedom to choose the type, content, and strategy for learning. That way of learning has something in it. Can we bring these positive features of online learning to the classroom, even without use of online environment?

Can we bring these positive features of online learning to the classroom even without use of online environment?

Example 2. The web site of CASMI (http://www.umoncton.ca/casmi, see Figure 5) was also demonstrated in this session. CASMI web site provides math problems to solve (i.e., CASMI community, Problem of the Week model, Virtual Mathematical Marathon…), and

mathematics enrichment activities (i.e. math competitions, games, puzzles, etc.) like the Math Forum (Drexel University). It offers the opportunity for collaborative solution by using Google docs.

- To adjust the level of difficulty is important. It needs to be shared within community.
- Bring problem solving to the classroom. There were no resources in French.
- There are no tools for elementary schools. Students are asked to communicate math, explain math. They are looking into creating a discussion forum.
- Have a group of 25 enrichment kids (Grade 7-8) who will create problems for others.
- First there was just text, then audio, and then video. Kids are more involved with technology; they are meeting challenges. The intent is to prevent "pedagogical drop-outs."

Figure 5: CASMI web site.

During the robotics example, it was stated that not only the school has a blog, but each student has a blog, and the act of learning is much more complicated than what is done in school. It is what everybody in the world is doing. Online learning is public and worldwide.

If you limit school mathematics to only certain topics and certain activities, you limit the human spirit. Technology can help go beyond the limits of human being.

Fourth meeting. Saturday 1:30pm - 3:15pm (Dragana Martinovic)

We started this meeting by discussing mathematics learning and teaching for the Net Generation. Some of the questions we discussed are the following:

- Who are the Net Generation learners?
- What are their needs?
- How do they use technology for learning purposes?
- How to integrate video and podcasting in the mathematics classroom and beyond it?

Example 1. A videoconferencing event from Martinovic's project *Teachers as ICT Agents* (http://www.uwindsor.ca/ictagents, see Figure 6) was presented to participants, and the participants reflected on it. This was a large scale event organized by one of our contributors, Timothy Pugh from the Grand Erie District School Board (http://brantford.com/index.cfm?page=home&section=news&Id=883). More than 50 teachers and 1200 students participated in a learning event in which the Canadian Space Agency was showcased. The expert from the Agency was in one of the participating schools while the other schools were connected over the videoconferencing units. In preparation for this event, students and teachers selected the best representative questions from Grade 4-12 curricula to ask the expert. The event was well received by the students. Some teachers stated that the event was not interactive enough, that the children were not paying attention all the time, and that the expert might have been better instructed on how to reach children at different levels of schooling.

The workshop participants provided the following comments:

- The point is in reaching experts.
- Net generation likes videos. We lose a lot of things when we just listen.
- Young people need to learn to focus and grasp information.
- Research points that top achievers do not use science or know how to do research.
- We need not to assume that technology will always be good. The fact this event was organized is valuable. Technology lets us take risks.
- There is math here. This event develops curiosity. It creates spirit of belonging.
- Things happen with technology. Technology is not always elitist-it may help reach those who do not have.

Figure 6: Students watching a presentation of an expert from the Canadian Space Agency, followed by the interactive question/answer period.

Things happen with technology. Technology is not always elitist-it may help reach those who do not have.

Example 2: The next example was from the Fields` website (developed by George Gadanidis) *Windows into Elementary Mathematics* (http://www.fields.utoronto.ca/mathwindows/, see Figure 7). The workshop participants commented:

- *Windows into Elementary Mathematics* looks like an interactive text book. However, since there are not enough online resources for mathematics, there is a need for all types of resources.
- University mathematics and school mathematics are very different. The question is, why are kids attracted differently by the same information given in different media?

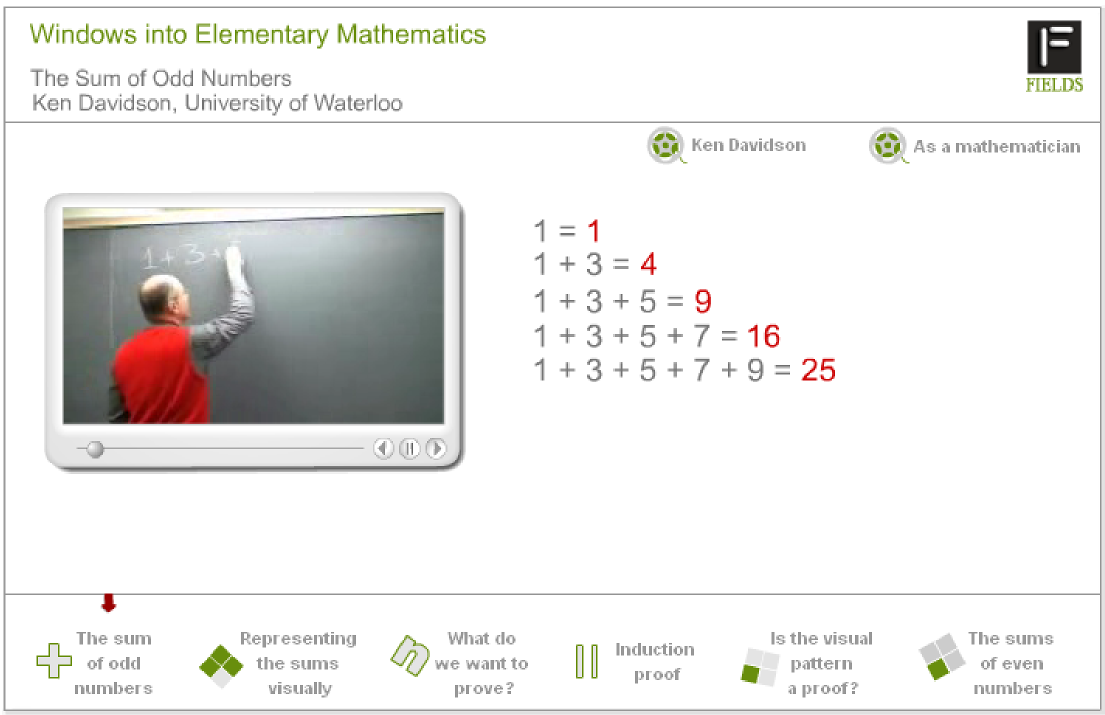

Figure 7: Windows into elementary mathematics: The sum of odd numbers.

Example 3: *Digital Windows into Mathematics* is a website (developed by George Gadanidis) covering presently five mathematics topics enriched with videos, songs, and a collaborative annotation tool (http://www.edu.uwo.ca/dwm/, see Figure 8)

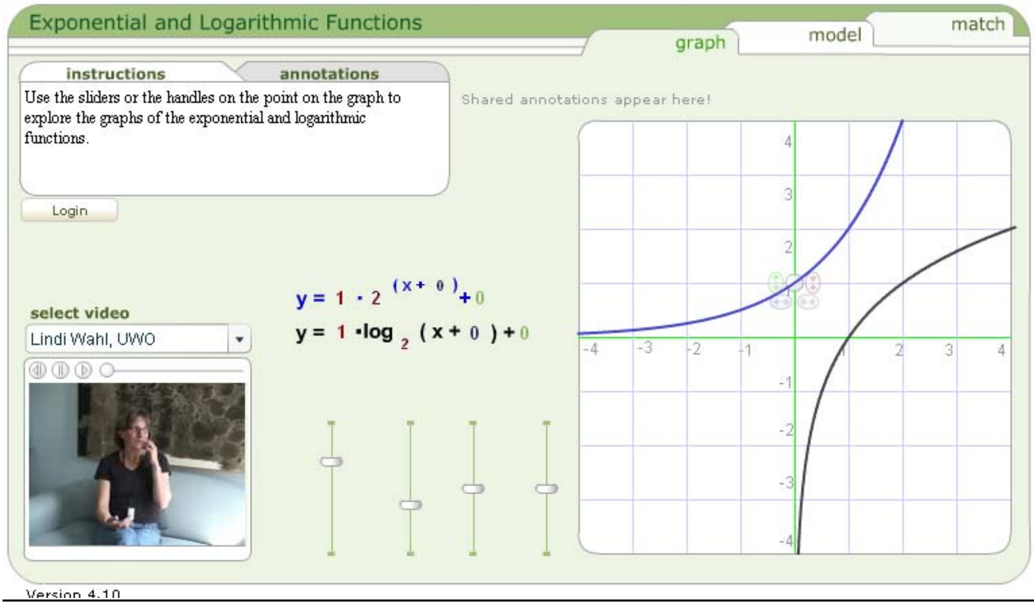

Figure 8. Digital Windows: Exponential and logarithmic functions

- This looks like being suitable for more advanced learners. Who are the learners?
- For all these objects, it is important how we use them.

<u>Example 4</u>: *Moebius on YouTube* (http://www.youtube.com/watch?v=JX3VmDgiFnY, see Figure 9).

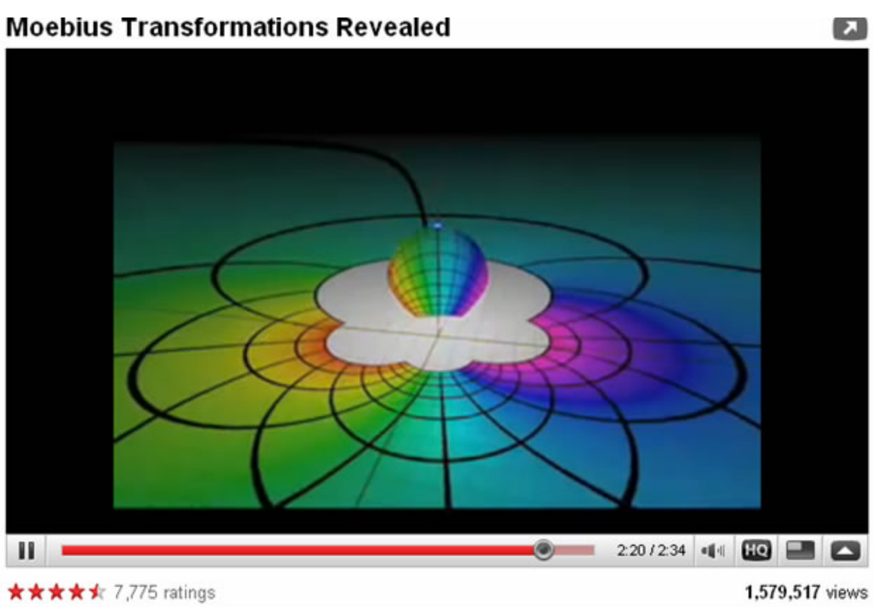

Figure 9: Moebius Transformations revealed on YouTube

- Over 1,500,000 individuals watched Moebius on YouTube. There is no any other way that these concepts would be able to reach and affect so many people.

> Power of YouTube—There is no any other way that these [mathematics] concepts would be able to reach and affect so many people.

What Do Teachers Say?

Prior to the CMEF2009 meeting, the facilitator asked some teachers who could not attend the meeting to answer questions regarding their experiences with online learning. In particular, the teachers were asked:

> What online formats did you use with your math students? Describe your students; describe both hardware and software used and their usability for math learning and teaching.
> Write about your math teaching experience with any of the following (or something else that I did not think of):
> -using online white boards (i.e., www.scriblink.com) for small groups to communicate mathematically and work together synchronously
> -using application sharing software (i.e., Elluminate Live!, Adobe Connect) and mathematics software on participants' computers to support synchronous work
> -using communication software (i.e., e-mail, discussion boards, blogs, wikis) to permit students to asynchronously discuss mathematics and share products by attaching files from mathematics software
> -linking wikis to online learning objects to permit users to discuss their actions on and ideas drawn from the objects
> -expanding the above by using learning objects that retain the manipulations of a user and permit these adjusted objects to be shared via a discussion tool (wiki, blog, discussion board)
> -describing the features of an online learning environment to explore mathematics
> -assessing work done online
> -record the interactive use of SmartBoards in the mathematics classroom and make expansion to the Web
> Provide a list of resources would help too (recommendation of what you like).

Three elementary and one secondary school teacher shared their experiences with us. When we analyzed their responses to our questions, we found the following themes important.

Technological issues

Technological issues emphasized by teachers vary from compatibility issues to the lack of necessary software. Some problems related to software could have been solved by using free and online tools which are available for educational purposes. Given that they lack the availability of these tools, teachers need more support to overcome these challenges through the in-service trainings.

- The tools to run an online course with my students were unavailable.
- There was no way to submit assignments or send assignments.
- Resources at the school were limited to the OSAPAC software installed on school computers. Many of the programs are game types for the primary, junior levels. Other than Spreadsheet software, Geometer's Sketchpad was really the only software used by some teachers and rarely used below intermediate grades.
- Many teachers themselves are uncomfortable with teaching using technology.
- There are compatibility issues with the operating system and therefore the instructions in the math textbooks did not match the version installed on some of the machines.
- Our archaic computers are unable to load the program…..my personal computer is the only one able to load, so I use it to demonstrate.
- We have 40 minutes once a week of available school lab time with compatible computers.
- Time limits were set with the students as to when they could be online and a help discussion area was set up so that students could ask questions and other students could go on and help each other.
- We have dealt with some of the compatibility issues by having documents in .rtf or .pdf format so that students can access the information from home. As for software, there are options of using Google docs for spreadsheets in mathematics,

however, students need an email and at this time, in the elementary panel, we are not recommending its use.

- We also have a Google educational application which allows students to access Google docs without using an email. This is only feasible if the students have Internet access from home. Geometer's Sketchpad is one of the few mathematical software programs that we are able to give to students for home use.

> Our archaic computers are unable to load the program…..my personal computer is the only one able to load so I use it to demonstrate.

Pedagogical issues

Teachers are aware of advantages and disadvantages of online learning although some of their understanding is based on assumptions. It is quite obvious that they need more training on how they and their students could benefit from these environments and more time to share their experience with their colleagues. This peer interaction may help them learn better the ways to use online environments effectively.

Another concern spelled out by teachers is to have weak students in these online programs. One reason for this could be that only students who do not have chance to take face-to-face course or who failed in the face-to-face course take the online course.

- The newer math textbooks have allowed teachers to expand on their math by giving the students a step-by-step approach to completing an assignment and then the students are to analyze their results. This required little knowledge on the teachers' part as to how the software works.
- Teachers become outdated by updated versions of the software. The issue of teacher training cannot be dismissed. There is still a gap between teacher training and their comfort with technology and the tools available to be used to deliver curriculum. The gap increases for those teachers not comfortable with technology and having to deliver the math curriculum using the tools that they are not familiar

with. Most resort to software that delivers the math in a game format as students usually pick this up quite quickly.

- Students who took online course weren't strong students to begin with and the online environment made the math tougher. They were weak math students who were clearly only used to chalk and talk math classes.
- I believe that the best use for the online learning environment is as an enhancement to a regular face-to-face course.
- I use the Smart board in my Grade 1 classroom quite often (we have two in our school that we share so it is not a permanent fixture in my classroom). I have used it for all of the math strands. The visuals are great….
- I use computer quite extensively with students for assessment "show me what you know."
- During my online courses, I had e-mail, discussion boards, blogs, wikis, embedded in the course but they did not really do them (and thus lost any marks associated with them).
- In my senior classes (mcv4u, mdm4u and map4c) I have many assignments that are done electronically and then I mark the files and send them back to students.
- Students are much more involved in the assignment when they are using the computer. They would rather use online manipulatives when given the choice between online and three-dimensional manipulatives. The activities are less foreign to them and are considered more as belonging to their generation.
- Students with special needs can also benefit from the online component as they are able to make and correct their mistakes easily, can work at their own pace without the rest of the class waiting and they can answer questions online with more confidence because they can check their answers before posting.
- For a teacher, tracking assignments is much easier. There are less "lost" assignments. Depending on the software being used, there can be quicker response time for student evaluation and students can benefit from the quick feedback. Programs also offer remediation based on the specific needs of that particular

student. This can be a huge time saver for teachers planning for the different needs of the class.

- Parents can also be more involved in their students' learning. The parents currently have access to their child account at our school and are encouraged to go on from time to time and check their students' assignments and marks. In mathematics, parents will have the advantage of viewing the lessons and using the tools the students are using. This encourages more parental involvement with students' homework. It also keeps the student more accountable for their assignments.
- Online technology allows those students that need an extra moment to think things through or have issues with shyness to participate more fully in lessons.

> Teachers become outdated by updated versions of the software. The issue of teacher training cannot be dismissed.

Issues of Access

Issues related to access are related to either technological issues or privacy issues. Some schools and school boards limit the use of the Internet because they are concerned about privacy of their students and uncontrolled interaction between students and teachers.

- I do not have access to any online devices, network restrictions are in place.
- This is still an issue with the Unions as they are hesitant to have teachers communicate with students online in any format. Teachers are also hesitant about the communication with students and also the boundaries of work and home.
- The students very easily learn how to use and interact with the SmartBoard and it is wonderful for my students who are kinaesthetic learners.

- I use a webpage for my classroom. I post homework and notes for both students and parents. I post pictures of activities and suggestions for lesson extensions. At this time it is not interactive but someday I would like to add that component to it.
- I just got the IT department to release to me individual student email accounts. They were very hesitant to do so. This is done more freely with secondary school students in our board.
- The email feature was restricted to each individual school. We had students in some schools sending inappropriate emails to students in other schools. Students were told that I would not be replying to emails. Until the issue was resolved with the union and a protocol set up, email communication with students was to be limited.

I do not have access to any online devices, network restrictions are in place.

Administrative issues

- Our board does not actively endorse online teaching of elementary students in any subject area. The board recently just started e-learning courses for secondary students.
- Time has been an issue in our school due to the large size. We are given only one 40 minute lab period per week. Since our lab period is Monday, we lose classes due to the holidays.
- Some more positive aspects of online learning in the elementary grades is the level of engagement for the students, paperless system which alleviates organizational issues assignment tracking, and parental awareness of student work.

Recommendations

Although there are still some challenges with providing effective and sustainable opportunities for online learning, some themes that emerged through discussion and teacher feedback seem promising. Visual and dynamic characteristics of online learning environments seem to be appreciated because these environments provide students with learning opportunities appropriate to their generation.

Indeed, it is observed that students, as Net Generation, use technology more than we expect. In some of the cases described by teachers, student use of technology has been inhibited for security and privacy concerns. However, these concerns could be overcome by employing appropriate technologies.

Therefore, one recommendation raised through the discussion done in our working group is that students should be encouraged to use contemporary educational tools to explore mathematical objects and ideas as well as the relationships among them. These tools could vary from various videos created to promote students motivation (see Math Performances at http://www.edu.uwo.ca/mpc/festival.html), explorative environments such as Math Towers (www.math-towers.ca), to visual online mathematical environments such as GeoGebra. Moreover, communication tools such wikis and blogs and public sharing environments such as TeacherTube (www.teachertube.com) could be introduced to the students, and students could be encouraged to "play" in these environments.

GeoGebra Institute of Canada

An important and exciting decision made in the CMEF meeting is to establish the Canadian GeoGebra Institute. We decided to promote GeoGebra in Canada because GeoGebra offers an online, free, and easy-to-use environment to create mathematical objects such as functions and geometric figures and to explore mathematical concepts and the relationships among these objects.

GeoGebra

Moreover, since GeoGebra is an open source tool, volunteers from many countries help translate it to their own languages. This makes GeoGebra suitable for Canadian multicultural environment. Online access and offer in multiple languages make GeoGebra, and through it mathematics, accessible to many students.

The role of the GeoGebra Institute of Canada will be to provide workshops for teachers and students, to provide learning guides for GeoGebra and dynamic worksheets for GeoGebra users. Furthermore, this Institute will organize a Canadian GeoGebra competition to promote the use GeoGebra among students.

The first attempt to create a GeoGebra community was to organize an ad-hoc session in the Canadian Mathematics Education Study Group (CMESG) meeting whish was held at York University in May 2009. We introduced GeoGebra to the participants, explained our action plan to establish the Canadian GeoGebra Institute, and invited them to be part of this new community.

Conclusions

As described earlier, the intent of this working group was to focus on recent developments in online learning and to investigate their usability for collaboration and exploration in teaching/learning mathematics. In hindsight, the group addressed larger scope of issues and put into practice some of the recommendations resulting from the previous CMEF (see Figure 1). Some recommendations and results of our work are presented in Figure 10. This figure illustrates activities that will be taken by the GeoGebra Institute of Canada, in terms of promoting online learning; teacher training; building repository of resources; and promoting open access, and collaborative and exploratory online activities in mathematics education. Research is another activity the Institute will take an active role in. The statement that we will “research what happens when students, teachers, technology and mathematics meet online” is a paraphrase of the Galbraith’s call for research that will further our “understanding of what happens when students, mathematics, and technology meet” (Galbraith, 2006, p. 287). Rephrased, this statement is better aligned with our goals and our appreciation for the role teachers have in adoption of online learning in schools and the issues they face in this process.

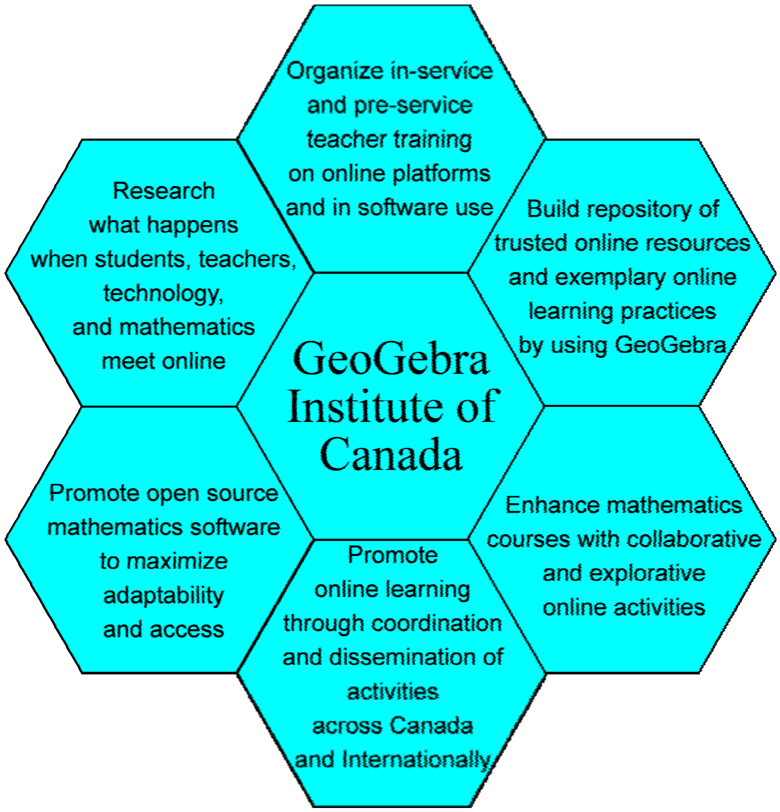

Figure 10. Activities of the GeoGebra Institute of Canada that are aligned with those proposed by the CMEF 2001 and 2009 meetings on online learning.

A lot has changed in the seven years between the two Canadian Mathematics Education Forums held in 2002 and 2009. The number of online courses and online degrees offered has plummeted. Continual development of Web possibilities supports collaboration and creativity. Mathematics resources on the Web are many and the education community recognizes their value for enhancing both formal and informal learning of mathematics. Advantages of computer networks for connecting schools and individuals are recognized in current curricula. However, some issues still remain unresolved: Teacher training in use of technology for learning mathematics, regular upgrade and replacement of outdated hardware and software in schools, development of assessment tools for online learning, and providing access in all schools to connective and collaborative technologies (e.g., Adobe Connect, video-conferencing units). There are other challenges as well. Technology is an enabler, but the real change is brought about by teachers, students, administrators, and researchers. As Galbraith wrote, “The challenge continues to grow” (2006, p. 287).

Resources

Fathom/Tinkerplots (OSAPAC)
GSP (OSAPAC)
Cabri 3d (OSAPAC)
Curve expert (http://curveexpert.webhop.biz/)
Winplot (http://math.exeter.edu/rparris/winplot.html)
Captivate (http://tryit.adobe.com/ca/captivate/?sdid=EQFSA)
Examview (http://www.swlearning.com/examview/examview_main.html)
Wetpaint (used to create Wiki sites, http://www.wetpaint.com/)
GeoGebra (www.geogebra.org)
Math Towers (www.math-towers.ca)
TeacherTube (www.teachertube.com)
PBWiki (www.pbwiki.com)

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